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USING THE FINITE ELEMENT METHOD TO ANALYSIS OF FREE VIBRATION OF THIN ISOTROPIC OBLATE SPHEROIDAL SHELL

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ABSTRACT

This paper focuses on the free vibration analysis of thin isotropic oblate Spheroidal shells. The shell properties are assumed to vary continuously through the thickness direction. The analysis is performed following the Finite Element method used the finite element package ANSYS. This method as well as to estimate the natural frequencies and mode shapes for the shells. The obtained results it is found that increase the natural frequencies with increase the thickness of shell and at constant the thickness of base with varying thickness of head the natural frequencies increase.

1. INTRODUCTION

The study of free vibration of spheroidal shell takes a considerable at in the published literature. Several investigators, using a variety of mathematical techniques, have obtained approximate solution for the natural frequencies of axisymmetric vibrations of thin oblate spheroidal shells. De Maggio and Silibiger [1] obtained a solution for the torsional vibrations of thin prolate spheroidal shell in terms of spheroidal angle functions. Kalnins was concerned with the vibration analysis of spheroidal shells, closed at one pole and open at the other, by means of the linear classical bending theory of shells [2]. Frequency equations are derived in terms of Legender function with complex indices, and axisymmetric vibration of the natural frequencies and mode shapes are deduced for all opining angles ranging from a shallow to closed spherical shell. It was found that for all opening angles the frequency spectrum consist of two coupled infinite sets of modes that can be labeled as bending (or flexural) and membrane modes. It was also found that membrane modes are practically independent of thickness, whereas the bending modes vary with the thickness. The same author concerned with a theoretical investigation of the free vibration of arbitrary shells of revolution by means of the classical bending theory of shells. A method is developed that is applicable to rotationally symmetric shells with meridional variations (including discontinuities) in Young's modulus, Poisson's ratio, radii of curvature, and thickness. The natural frequencies and the corresponding mode shapes of axisymmetric free vibration of rotationally symmetric shell can be obtained without any limitation on the length of the meridian of the shell. The results of free vibration of spherical and conical shells obtained earlier by means of the bending theory. In addition, parapoloidal shells and sphere-one shell combination are considered, which have been previously analyzed by means of the inextentional theory of shells, and natural frequencies and mode shapes predicted by the bending theory are given. Numergut and Brand [3] determined the lower axisymmetric modes of prolate shell with five values of eccentricity. DiMaggio and Rand [4] using membrane shell theory in which the effects of bending resistance are ignored. Their work was distinguished by applying their solution to constant thickness membrane shell by means of integrating numerically the equations of motion. It was found that the frequencies associated with higher modes are strongly dependent on the eccentricity ratio.

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Fawaz [5] Raleigh variation method was used to obtain natural frequencies and mode shape of axisymmetric vibration of thin elastic oblate spheroidal shells and presents the results of both theoretical and experimental investigations of the shell. The experimental investigation involves construction two closed form oblate spheroidal shells with two different eccentricities. Zhu [6] based upon general thin shell theory and basic equations of fluid-mechanics; the Rayleigh-Ritz's method for coupled fluid-structure free vibrations is developed for arbitrary fully or partially filled in viscid, irrigational and compressible or incompressible fluid, by means of the generalized orthogonally relations of wet modes and the associated Rayleigh quotients.Aleksandr Korjanik et,al [7] investigated the free damped vibrations of sandwich shells of revolution. As special cases the vibration analysis under consideration of damping of cylindrical, conical and spherical sandwich shells is performed. A specific sandwich shell finite element with 54 degrees of freedom is employed. Starting from the energy method the damping model is developed. Numerical examples for the free vibration analysis with damping based on the proposed finite element approach are discussed. Results for sandwich shells show a satisfactory agreement with various references solutions.
Linear and nonlinear vibrations of shallow spherical shells with free edge are investigated experimentally and numerically and compared to previous studies on percussion instruments such as gongs and cymbals[8].The next section is first devoted to an experiment al linear modal analysis of the spherical caps . It enables to make several comparisons, first with circular plates, and then with the gong studied in [9]. In the nonlinear regime, several combination of resonances are reported. They are found to be similar to those observed in gongs and cymbals.

This investigation deals with the free vibration of thin elastic oblate spheroidal shell. The shell is assumed be of isotropic material. The analysis depends on the Finite Element method.

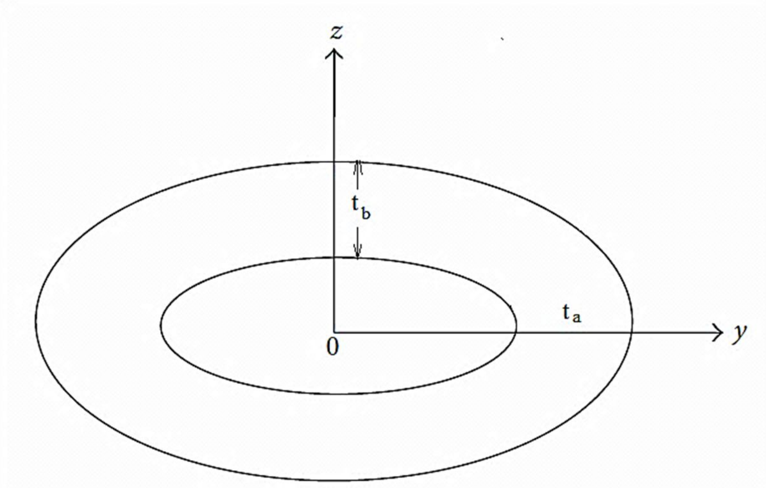

Figure 1 Oblate Spheroidal Shell Models

2. FORMULATION

2.1. Stress-Strain Relation-Ship

From the usual thin shell assumption, the normal stress σ_z is assumed small enough to be neglected and the corresponding ε_z is eliminated (plane stress problem is assumed) the stress- strain relations in coordinates aligned with principle material directions are given [1]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 \\ 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{1}$$

Where:

$c_{11}=c_{22}=(1-\nu)/A$

$c_{12}=c_{21}=(\nu)/A$

c_{44}=G

c_{66}=c_{55}=fG

$A=\frac{(1-\nu)(1-2\nu)}{E}$

$G=\frac{E}{2(1+\nu)}$

Where *f* is the shear factor for homogeneous shell should be given a value of 1.2 in order to account for the fact that the transverse shearing stresses produce too little strain energy [4].

2.2. Element Parameters

A quadratic element of quadrilateral shape consists of eight nodes, all of which are located on the element boundary have been used to define the shell finite element model. This type of element is used for shell structure applications for both membrane and flexure load conditions. In this section, the parameters that are concerned with the selected element are discussed. These parameters are basically included; the element property parameters include the material properties and the thickness of the element at each node. The formulation of such shell is based on three dimensional elasticity theory. It is assumed that the normal to the center-plane remain straight after deformation, but not necessarily normal to the center-plane. The ration of thickness value to the smallest element dimension must be equal or less than (0.1) in order to maintain the element to be thin [1].

The displacement at any point within the element is written in terms of nodal translations and rotations as:

$$\begin{Bmatrix} u(\xi,\eta) \\ v(\xi,\eta) \\ w(\xi,\eta) \end{Bmatrix} = \sum_{i=1}^{8} N_i(\xi,\eta) \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \sum_{i=1}^{8} N_i(\xi,\eta) rz \begin{bmatrix} a_i^u & b_i^u & c_i^u \\ a_i^v & b_i^v & c_i^v \\ a_i^w & b_i^w & c_i^w \end{bmatrix} \begin{Bmatrix} \theta_{xi} \\ \theta_{yi} \\ \theta_{zi} \end{Bmatrix} \tag{2}$$

Where:

$z = -\frac{t}{2} \rightarrow \frac{t}{2}$

N_i=Shape functions.

z = nodal thickness

u_i , v_i , w_i= global nodal displacements.

θ_{xi} , θ_{yi} , θ_{zi} = global nodal notations.

r = natural coordinate along thickness direction but normal to the shell surface.

a_i^u , a_i^v , a_i^w = direction cosines of u, v, and with respect to the nodal coordinate system at node i.

It is obvious that each node has six degrees of freedom, and then the element is of (48) degrees of freedom. Not all but some of the element degrees of freedom are considered at each of the finite element models, depending upon the function (boundary conditions) of that model.

2.3. Strain Displacement Relation Ship

The strain-displacement relationship can be written with its dependence explicitly expressed as:

$$\{\varepsilon\} = [B]\{u_i\} \tag{3}$$

All elements of the strain–displacement matrix, $[B]$, are derived in terms of the shape function derivatives and the Jacobian matrix.

2.4. Element Stiffness Matrix

In general, the basic concept of the finite elements method is to discrete the continuum into a definite numbers of small elements connected together at their common nodes. The strain-displacement matrix, $[B]$, as shown previously is given by:

$$\{\varepsilon\} = [B]\{u_i\} \tag{4}$$

And therefore; the element stiffness matrix can be written as:

$$[k]^e = \int_V [\mathrm{B}]^T [\mathrm{E}][\mathrm{B}]\, dV = \int_{-1}^{1} \int_{-1}^{1}\int_{-1}^{1} [\mathrm{B}]^T [\mathrm{E}][\mathrm{B}] \det\ [J]\ dr\ ds\ dt \qquad (5)$$

Where:

$[K]^e$: is the element stiffness matrix,

$|J|$: is the determinant of the Jacobian matrix.

2.5 Element Mass Matrix:

To derive the consistent mass matrix, one can consider the kinetic energy of the total solution domain is discretized into number of elements **(NE**) such that:

$$TI(\dot{u}) = \sum_{e=1}^{NE} TI^e(\dot{u}) \qquad (6)$$

Where *TI* and TI^e are the kinetic energies of the total solution domain and the sub–domain respectively. The kinetic energy of the element can be expressed as:

$$TI^e = \frac{1}{2}\int_V \{\dot{u}\}^T [m]\{\dot{u}\}\ \ dV \qquad (7)$$

The velocity vector within an element is discretized such that:

$$\{\dot{u}\} = \sum_{i=1}^{n} N_i \{\dot{u}_i\} \qquad (8)$$

n (number of nodes)

Thus, the mass matrix of the presented curved shell element is written as:

$$[M_1] = diag\ \ [[M_0]\ \ [M_0]\ \ \ [M_0]\ \ [0]] \qquad (9)$$

Where

$$[M_0] = \int_V \sum_{j=1}^{N_1} \rho_j \{N\}\{N\}^T dV \qquad (10)$$

Where, ρ_j is density of layer j. $\{N\}$is a vector of the shape functions for in plane motions involving only the nodal translations which are functions of ***s*** and ***t*** only. $[0]$ is a null matrix of size *24*24*. Eq.10 can be written in an expanded form as:

$$[M_0] = \int_{-1}^{1}\int_{-1}^{1} \sum_{j=1}^{N_1} \rho\ \ \int_{r_j^{bt}}^{r_j^{tp}} \det[J] dr\ \ \{N\}\ \{N\}^T ds\ \ dt \qquad (11)$$

3. FREE VIBRATION ANALYSIS

The free vibration analysis is the first step in the dynamic analysis, the natural frequency, ω , of the vibration is important to give an idea about the oscillation of the system with time, stiffness to weight ratios for different modes of oscillations, and to determine the natural period (*T*) of the vibration which represents the time for which the vibration repeats itself, as:

$$\mathrm{T} = \frac{2\pi}{\omega} \qquad (12)$$

Therefore, the free vibration (modal analysis) is used to determine the basic dynamic characteristic (vibration characteristic) of structures, which are the natural frequencies and mode shapes (normal modes). Natural frequencies and mode shapes are important parameters in the design of a structure under dynamic loading conditions. They are also needed if it is required doing dynamic analysis such as frequency, transient and spectrum analysis. To determine the natural frequencies of a structure, the governing differential equation of motion for the free vibration problem(no external applied loads) and undamped case is assumed in general [1].

$$[M_0]\{\ddot{X}\}+[K_1]\{X\}=\{0\} \tag{13}$$

Assuming harmonic motion that is:

$$\{X_i\}=\{\phi_i\}\sin\omega_i t \text{ ; i= 1,2,....k} \tag{14}$$

Where:

K= the number of $D.O.F.$ of the system
$\{\phi_i\}$=the mode shape vector for the i^{th} mode of vibration, and
ω_i = the angular frequency of mode i.

Differentiating Equation (14) twice with respect to time yields:

$$\{X_i\}=-\omega_i^2\{\phi_i\}\sin\omega_i t \tag{15}$$

Then, substituting Equations (14) and (15) into Equation (13) yields, after canceling the term ($\sin\omega_i$):

$$([K_1]-\omega_i^2[M_0])\{\phi_i\}\{\phi_i\}^T=\{0\} \tag{16}$$

Eq. 16 has the form of the algebraic eigenvalue problem$(K\phi=\lambda M\phi)$. From the theory of homogenous equations, nontrivial solutions exist only if the determinant of the coefficient matrix is equal to zero. Thus:

$$\left|[K_1]-\omega_i^2[M_0]\right|=\{0\} \tag{17}$$

Expansion of the determinant yields a polynomial of order NR called characteristic equation. The NR roots of this polynomial (ω_i^2) are the characteristic values or the eigenvalues. The cyclic natural frequency (f_i) is then obtained from:

$$f_i=\omega_i/2\pi \tag{18}$$

Substitution of these roots (one at each time) into the homogeneous Eq. 17 produces the characteristic vectors or the eigenvectors $\{\phi_i\}$ within arbitrary constants.

A number of solution algorithms have been developed for the solution of the eigenvalue problem. However, the inverse iteration method will present and used in this work.

4. RESULTS AND DISCUSSION

Fig. 3 shows the natural frequency (*Hz*) of the first three modes of vibration as a function of the thickness. It is shown that the increase of the natural frequency with increase the thickness, the reason of that the increase of stiffness for oblate with increase thickness of it.

Fig. 4 give the first four the natural frequency as a function of thickness ratio(t_a/t_b) for an oblate spheroidal shell have eccentricity ratio (*0.684*), where shown in this figure increase of the natural frequency with increase or constant the thickness of base and decrease thickness of head, because increase the stiffness of the oblate shell at the clamped region.

The reason of that behavior of this case when decrease the thickness of the head this tend to decrease the mass of the oblate shell and the stiffness of shell decrease but clamped region will stronger this week in this case.

Fig. 5 it is give the natural frequency as a function of the thickness ratio (t_b/t_a), this figure shown increase or constant the thickness of head and decrease the thickness of base at the support region this case explain the week of oblate shell, because when decrease thickness cause decrease the mass and stiffness of oblate and the support region don't stronger this week because smallest this region.

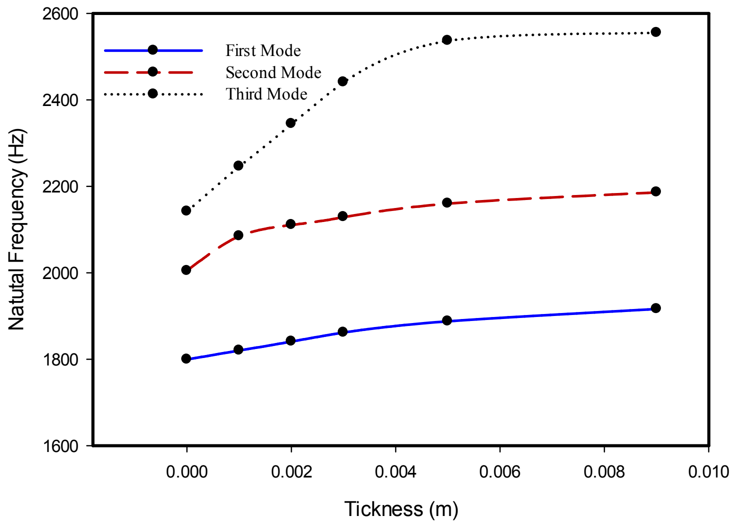

Figure 3 Natural Frequencies for First Three modes for Oblate Spheroidal Shell

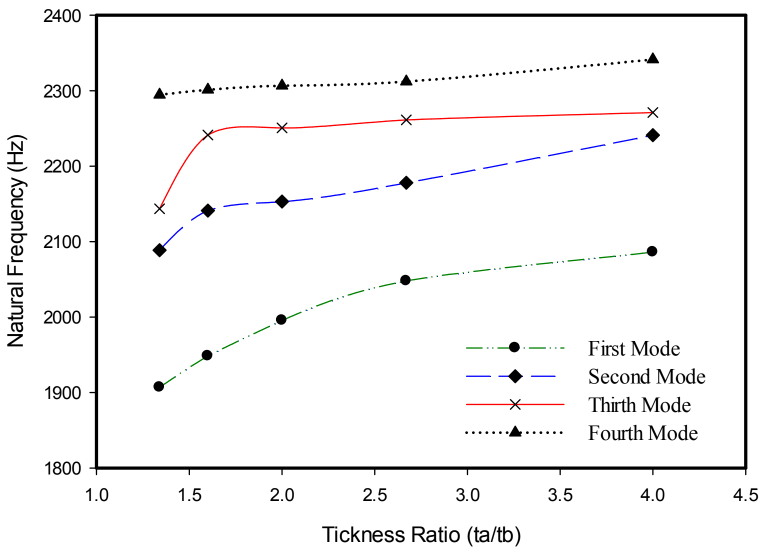

Figure 4 Natural Frequencies for First Fourth modes for Model (1).

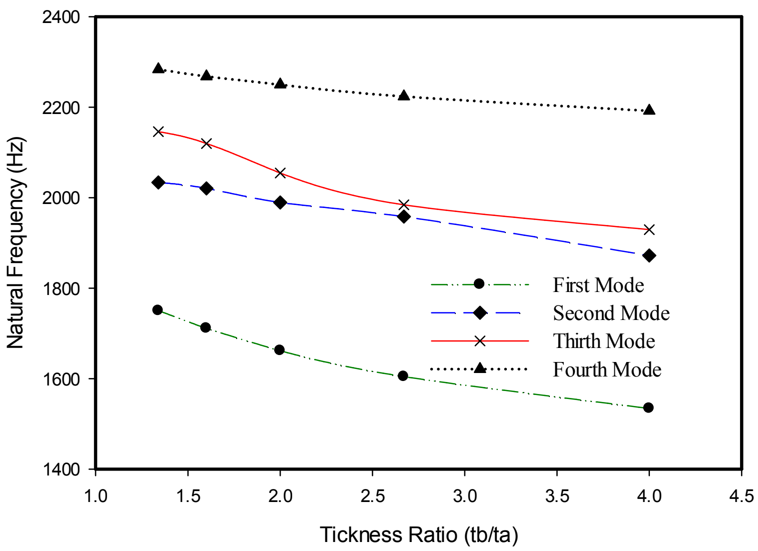

Figure 5 Natural Frequencies for First Fourth modes for Model (2).

Table 1 Theoretical and Experimental Natural Frequencies for Oblate Spheroidal Shell (e= 0.683 , a=0.185 m , b= 0.135 m , h= 0.0015 m)

Mode	Present work	Experimental work [10]	Nawal H.A, 2005 [11]
1	2394	2412	2517
2	2749	2635	2973
3	2832	2951	3086
4	3111	3126	3180

5. CONCLUSIONS

The main conclusions of the present work can be summaries as:

1. Natural frequencies are seen to have two types of behavior against increasing the thickness ratios (t_a/t_b) and (t_b/t_a), where increase with increase the ratio (t_a/t_b) and decrease with increase the ratio (t_b/t_a)
2. The natural frequency increases with increase the thickness of shell.
3. In case increase the thickness ratio (t_a/t_b) give strong structure of shell.

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